CHAPTER 12

SERIES-PARALLEL CIRCUITS

12.1 EQUIVALENT OR TOTAL RESISTANCE

The procedures for solving problems involving either series or parallel resistors are fairly clearly defined, but there are so many possible series-parallel combinations of resistors that it is difficult to apply a set pattern for determining the total resistance of a circuit. In general the best procedure to adopt is to first reduce single resistors in series or parallel to their equivalent value, then redraw the circuit and reduce it still further using the same procedure until a single equivalent resistance is obtained.

Example 12.1

Determine an equation for the total resistance in figure 12.1.

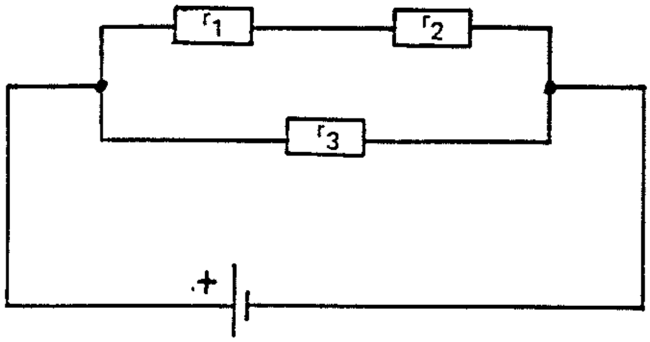


Figure 12.1

Reduce r_1 and r_2 in Figure 12.1 to single resistance value. (Figure 12.2).

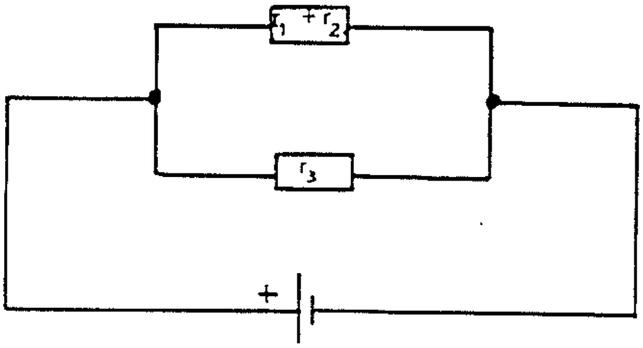


Figure 12.2

Reduce the parallel branch to a single resistor.

$$R_T = \frac{1}{\frac{1}{r_1 + r_2} + \frac{1}{r_3}}$$

Example 12.2

Determine an equation for the total resistance in figure 12.3.

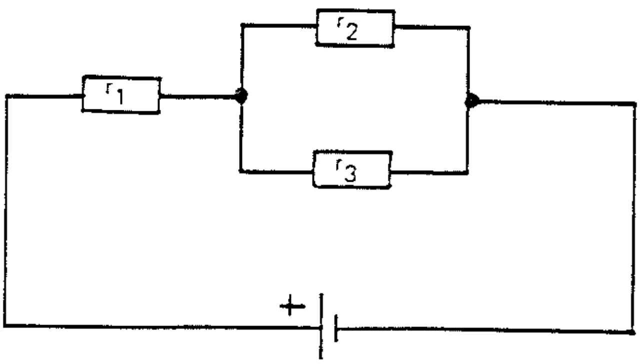


Figure 12.3

Reduce the parallel branch in Figure 12.3 to a single resistance value.

$$R = \frac{1}{\frac{1}{r_1} + \frac{1}{r_2}}$$

This reduces the circuit to a simple series circuit. (Figure 12.4 .)

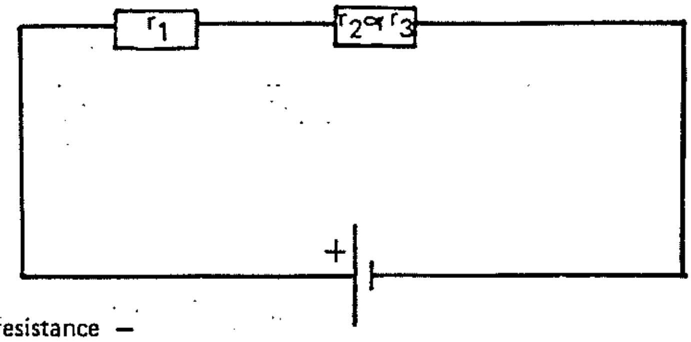


Figure 12.4

Then the total resistance –

$$R_T = r_1 + \frac{1}{\left(\frac{1}{r_2} + \frac{1}{r_3}\right)}$$

It can be seen from the two previous examples that the equation for calculating the total resistance is different in each case. Different series parallel combinations will always produce different equations for the total resistance. For this reason, the student is advised not to try to remember equations but to work systematically through a problem reducing the circuit in small steps until a final solution is reached.

Example 12.3

Calculate the total resistance of the circuit shown in Figure 12.5.

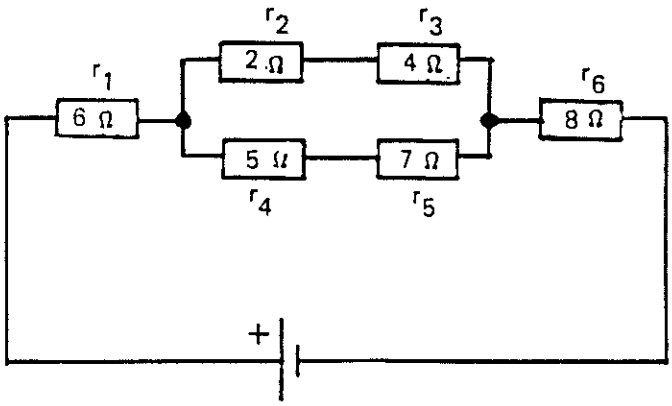


Figure 12.5

Collecting the series resistors inside the parallel branches reduces the circuit to figure 12.6.

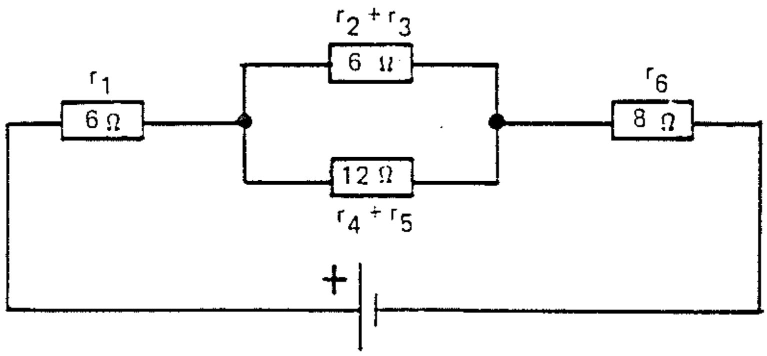


Figure 12.6

Convert the parallel branch to a single resistor.

$$R = \frac{1}{\frac{1}{6} + \frac{1}{12}}$$

$$= \frac{1}{\frac{2 + 1}{12}}$$

$$= 4\ \Omega$$

Then the circuit becomes figure 12.7.

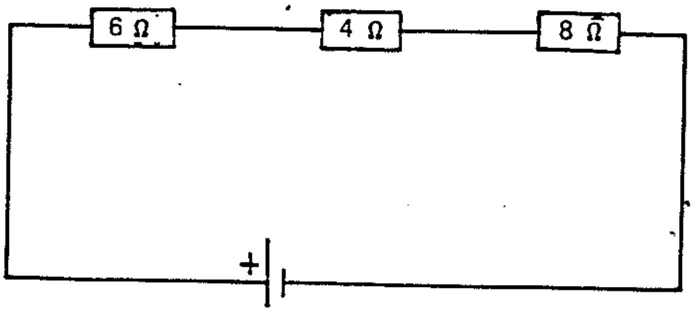


Figure 12.7

and adding the resistances in series gives –

$$R_T = 6 + 4 + 8$$

$$= 18\ \Omega$$

Example 12.4

Calculate the resistance of the circuit shown in figure 12.8.

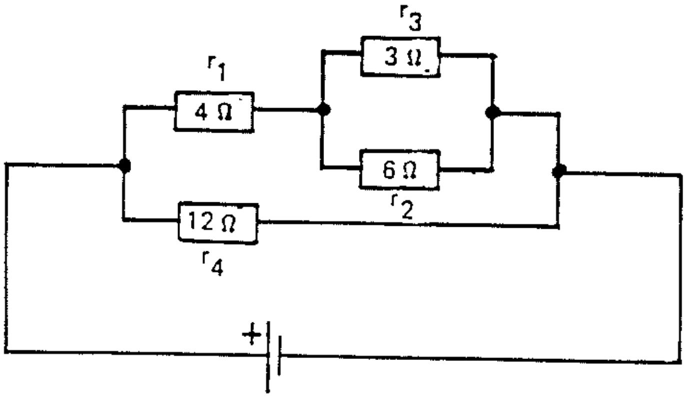


Figure 12.8

Simplify the parallel circuit containing r_2 and r_3 –

$$r_2 + r_3 = \frac{1}{\frac{1}{r_2} + \frac{1}{r_3}}$$

$$= \frac{1}{\frac{1}{3} + \frac{1}{6}}$$

$$= \frac{1}{\frac{2 + 1}{6}}$$

$$= \frac{1}{\frac{3}{6}}$$

$$= 2\ \Omega$$

The circuit is reduced to that shown in figure 12.9.

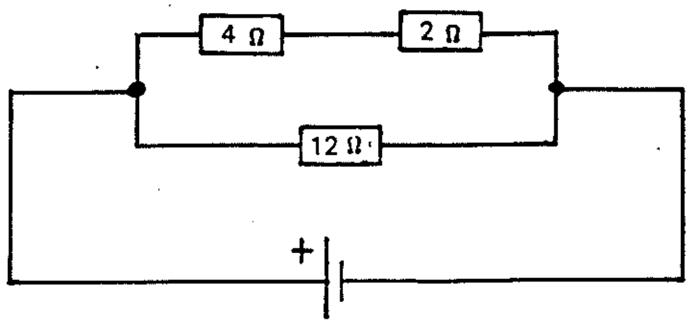


Figure 12.9

Add r_1 and $(r_2 + r_3) = 4 + 2$

$= 6\ \Omega$

This will give a simple parallel circuit. (Figure 12.10).

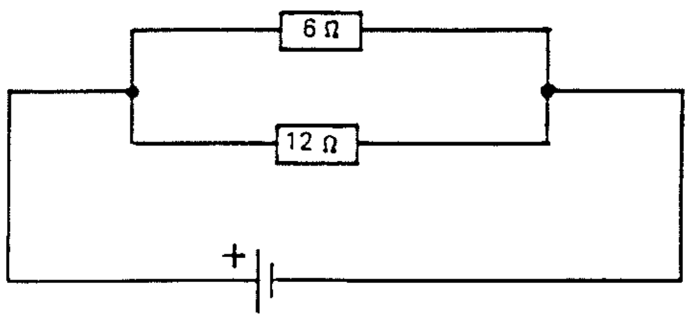


Figure 12.10

Calculate the equivalent value of the parallel circuit.

$$R_T = \frac{1}{\frac{1}{6} + \frac{1}{12}}$$

$$= \frac{1}{\frac{2 + 1}{12}}$$

$$= \frac{1}{\frac{3}{12}}$$

$$= 4\ \Omega$$

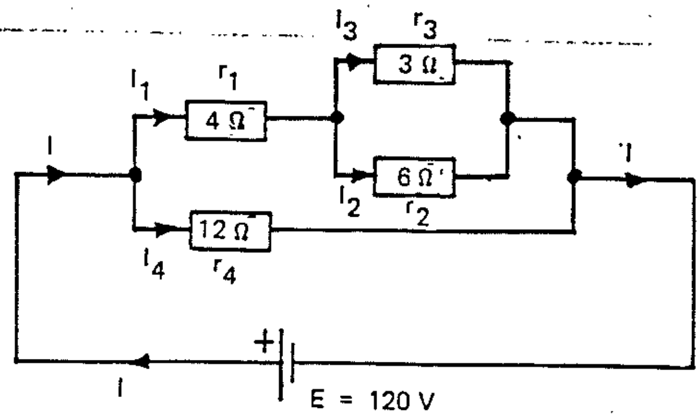
All series-parallel circuits may be reduced to a single equivalent resistance if the principles used in this chapter are applied to the circuit.

12.2 CURRENT AND VOLTAGES IN A SERIES-PARALLEL CIRCUIT

To determine the voltages and currents within a series-parallel circuit, the equivalent or total resistance of the group is first calculated and Ohm's Law applied to find the current drawn from the supply. The procedure is to then work from the supply back into the circuit, using the information gained in calculating the resistance, to assess the individual currents and voltages inside the circuit.

Example 12.5

An e.m.f. of 120 volts is applied to the circuit in example 12.4 Calculate all the branch currents and internal voltages drops of the circuit.



The total resistance of the circuit in example 12.4 was found to be 4 ohms.

$$I = \frac{E}{R}$$

$$= \frac{120}{4}$$

$$= 30A$$

From observing the circuit it can be seen that r_4 is connected directly across the supply voltage so I_4 can be immediately calculated

$$I_4 = \frac{E_4}{r_4}$$

$$= \frac{120}{12}$$

$$= 10A$$

If $I_4 = 10A$

$I_1 = 30 - 10$

$= 20A$

The voltage drop across $r_1 = I_1 r_1$

$= 20 \times 4$

$= 80V$

The voltage drop across the parallel section will be 120-80 = 40V.

$$\text{the current in } r_2 = \frac{E_2}{r_2} = \frac{40}{3} = 13.3\text{A}$$

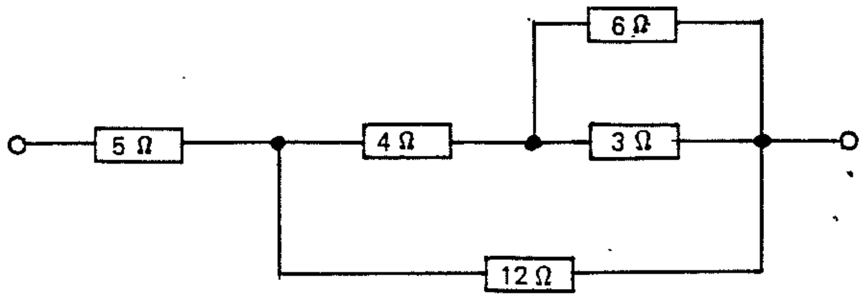
$$\text{the current in } r_3 = \frac{E_3}{r_3} = \frac{40}{6} = 6.6\text{A}$$

so the voltages and currents within the circuit will be:-

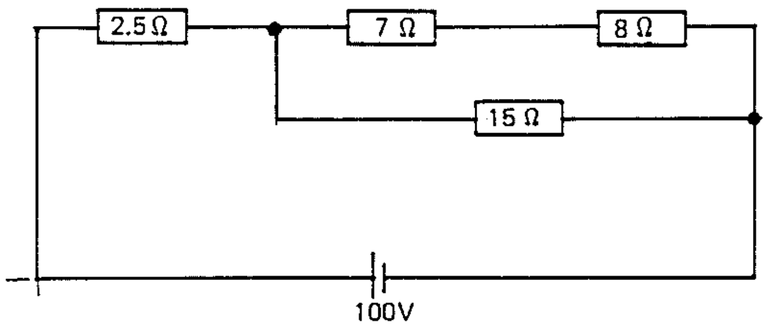
E	=	120V	I	=	30A
V_1	=	80V	I_1	=	20A
V_2	=	40V	I_2	=	13.3A
V_3	=	40V	I_3	=	6.6A
V_4	=	120V	I_4	=	10A

TUTORIALS 1.12

(1) Calculate the resistance of the following circuit.



(2) Determine the resistance and supply current in the following circuit.



(3) Calculate all the currents and voltages within the following circuit.